Math 2: Lines

Provided by Madrona Tutoring

Lines

You're going to see a lot of line equations (and things you could turn into lines if you wanted to) on the SAT. Let's break them down.

$y = mx + b$ This is slope intercept form. m is the slope, b is the y-intercept (which is where the line crosses the y-axis).

$Ax + By = C$ This is standard form. Slope is $-\frac{A}{B}$. The Y Intercept is $\frac{C}{B}$

Systems of Equations

A system of equations looks like this:

$2x + 3y = 26$

$x - 3y = 13$

A system like this is giving you two lines, and asking you where they intersect. For what set of coordinates (x,y) will both equations be true? There are three ways of dealing with a system of equations.

Method one: Elimination. When you eliminate, you treat the whole system like a big addition problem, and eliminate one of the variables (x or y) when you add everything up. This leaves everything in terms of a single variable that you can solve for, and then plug back into either equation.

1. $(2x + 3y = 26) + (x - 3y = 13) = (3x = 39)$
2. $\frac{x}{3} = \frac{39}{3}$
3. $x = 13$

If $x = 13$, then I can plug 13 in for x in one of the equations so solve for y.

1. $2(13) + 3y = 26$
2. $26 + 3y = 26$
3. $26 - 26 + 3y = 26 - 26$
4. $3y = 0$
5. $\frac{y}{3} = \frac{0}{3}$
6. $y = 0$

So, at the end we know that the solution to the two lines (where they cross on a graph) is at point $(13, 0)$.

Method Two: Substitution. This is when you can rearrange one of the equations so that x or y equals something that you can plug back in. This is best done with a system which is simple enough to make it easy. Again, this lets you solve for a single variable, and then plug your answer for that variable back into the original equation and solve for the other variable. Let's look at that same equation again.

$2x + 3y = 26$
$x - 3y = 13$

1. We need to solve for a single variable. The second equation looks easier, so let's use it: $x - 3y = 13$ can be rearranged as: $x = 13 + 3y$
2. Now we know what x is, so we can substitute $13 + 3y$ in for the x in the other equation to solve for y when $x = 13 + 3y$
3. With the substitution we see: $2(13 + 3y) + 3y = 26$
4. Now we simplify: $26 + 6y + 3y = 26$
5. Combine similar terms: $26 + 9y = 26$
6. Isolate the variable by subtracting the 26 from both sides: $26 - 26 + 9y = 26 - 26$
7. Simplify: $9y = 0$
8. Solve for y: $y = \frac{0}{9}$
9. $y = 0$

Okay, so we found out that $y = 0$ again. If we plug that in to one of the equations, we'll see that $x = 13$ again.

Method Three: Use your calculator. Press Y=, and plug the two equations in, and then you can either GRAPH them and check their intersection, or use the TABLE button to see where they intersect. If you do use the GRAPH button and don't see anything, check how much of the x and y axes are displayed.

Word Problems with Systems

Sometimes, you'll be handed a huge word problem with a bunch of variables in it. Usually you can make a system out of it to solve it.

For example: The YMCA is holding a New Years Bash, and everyone who brings their own noisemakers pay 3 dollars at the door, and everyone else pays 6 dollars. At the end of the night it was determined that 120 people attended, and a total of 480 dollars were raised. How many people brought their own noisemakers?

First, we set up our equations. One for money, and one for people.

$$3dollars(NoiseyPeople) + 6dollars(SilentPeople) = 480dollars$$

$$(NoiseyPeople) + (SilentPeople) = 120People$$

So, let's agree that $x = NoiseyPeople$ and $y = SilentPeople$. When we put in our variables it looks more like this:
$3x + 6y = 480$
$x + y = 120$
Then we solve using substitution or elimination just like above. How many people did bring their own noise makers?

Quadratics

The basic quadratic looks like this $Ax^2 + Bx + C = 0$. You'll encounter a few of them on the SAT. It's important to remember that to find out what x is in a quadratic equation, you have to factor it.

How do we factor? Well, lets start with this quadratic: $x^2 + 2x + 1$

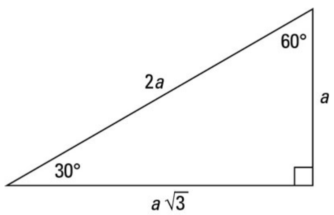
Triangles

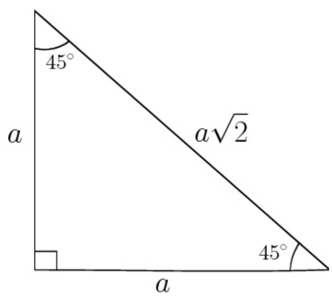
The basics: There are 180 degrees inside any triangle. The most common types of triangles that you will see on the SAT are right triangles (which contain one 90 degree angle), isosceles triangles (which have two congruent sides), and the special right triangles (30/60/90, and 45/45/90).

Area: $\frac{1}{2}bh$

Pythagorean Theorum: $a^2 + b^2 = c^2$

Special triangles: 30/60/90 and 45/45/90





Necessary Trigonometry

Remember SohCahToa? The good news is that for the most part, the SAT does one of two things with trig problems: either they only require a basic knowledge of SohCahToa, or they give you the formula you need.

Okay, so Sine, Cosine, and Tangent are all ratios, and just like the Pythagorean Theorem, they can only be used with right triangles.

$$Sine = \frac{opposite}{hypotenuse}$$

$$Cosine = \frac{adjacent}{hypotenuse}$$

$$Tangent = \frac{opposite}{adjacent}$$

In the most basic problems, they're give you two sides, and ask you to use the Pythagorean Theorem to find the third side to figure our either the Sine, Cosine, or Tangent.

1. For example: Given that in right triangle ABC $\sin(a) = \frac{3}{5}$, what is $\cos(a)$?